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Stable non-BPS bound states of BPS D-branes

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Abstract: S-duality symmetry of type IIB string theory predicts the existence of a stable non-BPS state on an orbifold five plane of the type IIB theory if the orbifold group is generated by the simultaneous action of $(-1)^{F_L}$ and the reversal of sign of the four coordinates transverse to the orbifold plane. We calculate the mass of this state by starting from a pair of D-strings carrying the same charge as this state, and then identifying the point in the moduli space where this pair develops a tachyonic mode, signalling the appearance of a bound state of this configuration into the non-BPS state.

Keywords: D-branes, String Duality.

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1. Introduction and summary

Quite often duality symmetries in string theory predict the existence of stable solitonic states in string theory which are not BPS states, but are stable due to the fact that they are the lightest states carrying a given set of charge quantum numbers. Several examples of this kind were discussed in ref.[1, 2]. A class of examples discussed in [1] involved a Dirichlet p-brane[3] on top of an orientifold p-plane[4]. This system has an SO(2) gauge field living on the brane and massive non-BPS states charged under this SO(2) gauge field. These states must remain stable in the strong coupling limit since there are no states into which they can decay. In [1] we identified these states in the dual weakly coupled theory for $p = 4$, 6 and 7. In this paper we shall focus on the case $p = 5$.

Since most of this paper involves details of calculations involving D-branes, we shall summarise the main results in this section. The orientifold 5-plane (O5-plane) in type IIB theory is obtained by modding out type IIB string theory in ten dimensional Minkowski space $R^{9,1}$ by $\Omega \cdot \mathcal{I}_4$, where Ω denotes the world-sheet parity transformation and $\mathcal{I}_4$ denotes the reversal of sign of four of the space-like coordinates. For a Dirichlet 5-brane (D5-brane) on top of an O5-plane, the state carrying charge under the SO(2) gauge field on the world volume arises from an open string state which starts on the D5-brane, goes once around the orientifold plane, and ends on the D5-brane. Two different orientations of the string give states of opposite charge on the world-volume, as shown in Fig. 1. (Although we are considering a configuration of coincident D5-O5 system, we shall often display them as a resolved pair for the sake of clarity.)

We shall normalise the charge in such a way that the states displayed in Fig. 1 carry charge 2 and -2 respectively.[1] We shall denote these states as $++$ and $--$ respectively. Besides these non-BPS states, there are other BPS configurations which carry charge under the world-volume gauge field. These are semi-infinite fundamental strings (F-strings) ending on the D-brane. There are four different configurations of fundamental strings which differ from each other by the orientation of the string, and the path followed by the F-string from infinity to the D5-brane. These four configurations are shown in Fig. 2 and carry charges ± 1 under the D-brane world-volume gauge field. From the point of view of the D-brane world-volume field theory these states have infinite mass due to infinite length of the semi-infinite F-strings.

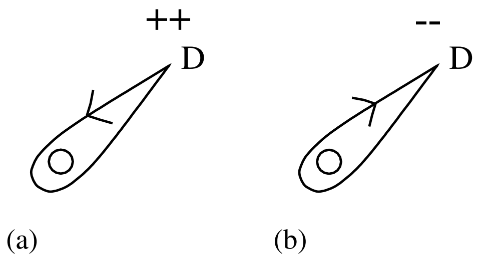

Figure 1: Open string configuration giving charged state on the world-volume of the D5-brane O5-plane system. The $\circ$ denotes the location of the O5-plane and the D denotes the location of the D5-brane.

In seeking a description of these states in the strong coupling limit, we go to the dual type IIB theory by making an S-duality transformation. Since S-duality transforms Ω to $(-1)^{F_L}$, where F_L denotes the contribution to the space-time fermion number from the left-moving sector of the world-sheet, we expect the O5-plane to get mapped to the fixed plane of the orbifold $R^{9,1}/(-1)^{F_L} \cdot \mathcal{I}_4$. However, as was argued in [5], the orbifold plane actually describes the dual of the coincident O5-plane $-$ D5-brane system. One way to see this is to note that the twisted sector in the orbifold theory contains a gauge field living on the orbifold plane. Since an isolated O5-plane does not have a gauge field living on the plane, the orbifold plane cannot be dual to an isolated O5-plane.

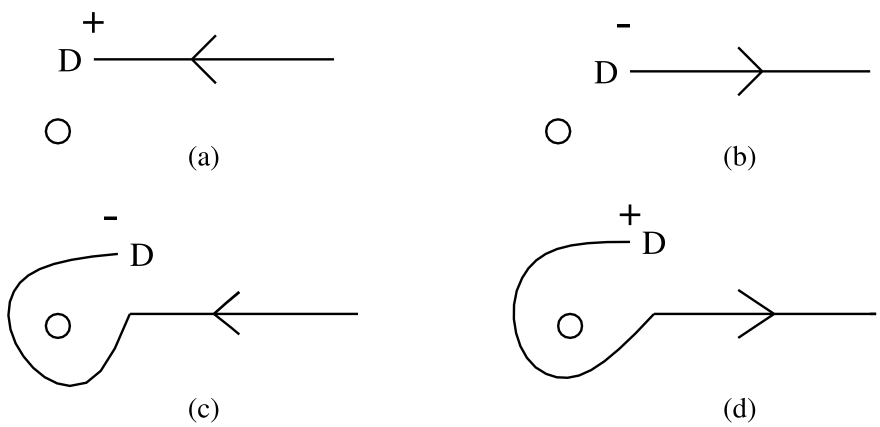

Figure 2: Four possible configurations of fundamental string ending on a D5-brane $-$ O5-plane system.

[1] This convention differs from the one used in ref.[1] by a factor of 2.

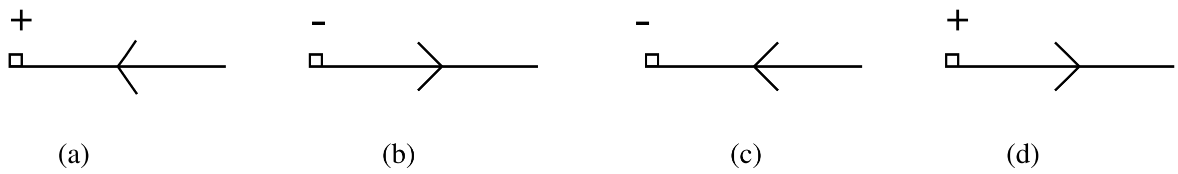

Figure 3: S-dual of the configurations shown in Fig. 2. Here the little square denotes the location of the orbifold plane.

On the other hand a D5 brane on top of an O5-plane has precisely the same degrees of freedom as those appearing on the orbifold plane.

If this identification is correct, then the orbifold plane must support configurations which are S-dual to the configurations shown in figs. 1 and 2. Configurations dual to 1 are the states carrying charge ± 2 units under the (appropriately normalised) twisted sector gauge field living on the orbifold plane. We shall refer to these as $++$ and $--$ states respectively. On the other hand the dual of the configurations in Fig. 2 are D-strings ending on the orbifold plane, as shown in Fig. 3. The ends of these strings should carry ± 1 unit of charge under the twisted sector gauge field. Note that for each orientation of the D-string, we must find two states, carrying opposite charges under the twisted sector gauge field.

Our first task will be to find appropriate conformal field theory description of these BPS configurations, which we shall do in section 2. The construction is best described by using the boundary state formalism discussed in ref.[6] (following earlier work of [7, 8, 9]) in which to each D-brane we associate a coherent state in the closed string sector describing the wave-function of a closed string emitted from the brane. (For other applications of this formalism see [10]. Application of the boundary state formalism to D-branes moving on orbifolds has been previously discussed in [11].) The allowed set of D-branes in the theory are then classified by classifying the possible consistent boundary states satisfying appropriate symmetry requirements. In our analysis we find precisely four possible consistent boundary states, corresponding to the four D-string configurations shown in Fig. 3. The existence of these BPS configurations provide further support to the conjecture that the orbifold plane describes the dual of a D5-brane on top of an O5-plane.

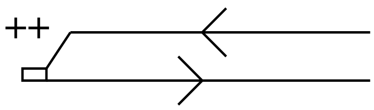

Figure 4: Superposition of the configurations shown in Fig. 3(a) and (d).

Once we have constructed the boundary state describing these elementary configurations, we can also superpose them. In particular, if we superpose the configurations given in Fig. 3 (a) and (d), we get a state carrying charge 2 under the twisted sector gauge field. This configuration is shown in Fig. 4. The Ramond-Ramond two form charge, carried by the individual D-strings, cancel between the two strings. Thus this state has the same quantum number as the S-dual of the state displayed in Fig. 1(a). However this still has infinite mass from the tension of the two semi-infinite D-strings and hence cannot be interpreted as a state living on the orbifold plane. Furthermore this system has a tachyonic field living on the D-string world-volume from open strings stretched between the D-string and the anti-D-string and hence is expected to be unstable. We interprete this instability as being due to the fact that the D-string anti D-string system of Fig. 4 is unstable against decay into a single ++ state living on the orbifold plane.

It is tempting to speculate that the ++ state can actually be regarded as the state of the D-string anti-D-string pair displayed in Fig. 4 after tachyon condensation. If T_0 be the value of the tachyon that minimizes the potential, then far away from the orbifold plane the tachyon field must approach the value T_0. On the other hand the analysis of section 2.3 tells us that the tachyon field must vanish on the orbifold plane. Thus the minimum energy configuration will be given by a tachyon field configuration of the form shown in Fig. 5. If this configuration is to describe the ++ state, then the negative vacuum energy density on the D-string due to tachyon condensation far away from the orbifold plane must exactly cancel the contribution coming from the D-string tension[2] since otherwise this will give infinite mass. If this conjecture is correct, then the mass of the ++ state, measured in string metric, will be of the form $C/(g\sqrt{\alpha'})$ where g is the string coupling constant, and C is some numerical constant. Unfortunately, in the absence of exact knowledge of the tachyon potential we cannot verify the validity of this conjecture, or proceed further along this line to calculate the numerical coefficient C.[3,4]

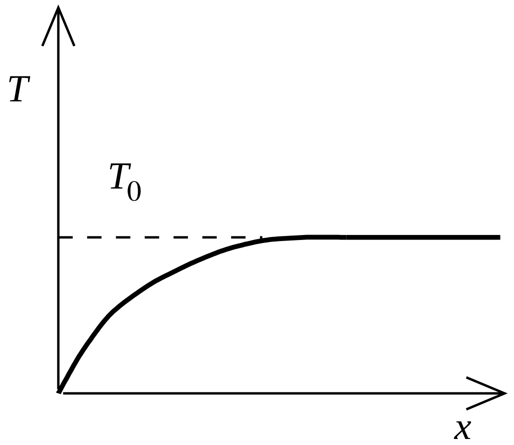

Figure 5: The tachyon field on the D-string - anti-D-string pair as a function of distance x from the orbifold plane.

In order to gain further insight into this problem, we consider in section 3 type IIB string theory on an orbifold $R^{8,1} \times S^1/(-1)^{F_L} \cdot \mathcal{I}_4$ where $\mathcal{I}_4$ reverses the sign of the coordinate on S^1 and three other space-like directions. This orbifold has two fixed planes, separated by a distance πR where R is the radius of the circle. As shown in Fig. 6, this system is expected to have eight different BPS configurations corresponding to D-strings stretched between the two orbifold planes. We explicitly construct these D-brane configurations by finding the appropriate boundary state in the closed string sector for each of these brane configurations. Each of these configurations has mass $\pi R T_D$ where T_D is the D-string tension.

[2]This in turn implies that space-time supersymmetry is restored fully in the bulk of the D-string anti-D-string pair after tachyon condensation. Examples where supersymmetry is restored as a result of tachyon condensation have been discussed previously in refs. [12, 13].

[3]If instead we consider the superposition of configurations in Fig. 3(a) and 3(b), then the boundary condition on the tachyon near the orbifold plane requires its derivative along the string to vanish. Thus the lowest energy configuration corresponds to $T = T_0$ everywhere. According to our conjecture this

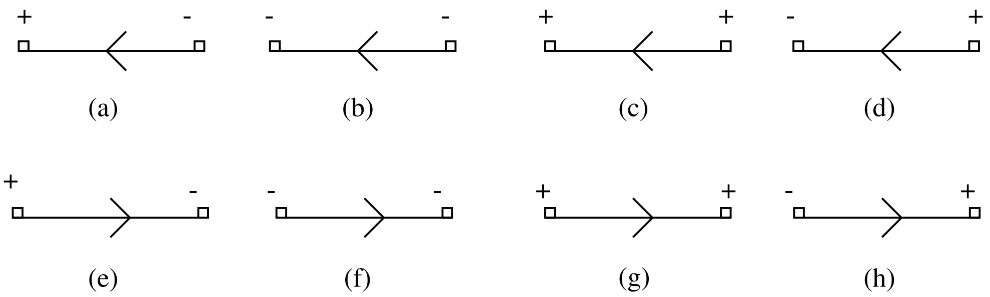

Figure 6: The eight different configurations of D-strings stretched between the two orbifold planes in type IIB on $R^{8,1} \times S^1/\mathcal{I}_4 \cdot (-1)^{F_L}$.

We can also find the boundary states corresponding to the superposition of any number of these D-branes by superposing the corresponding boundary states. Let us consider the superposition of the configurations in figs. 6(a) and 6(g). The corresponding state is shown in Fig. 7 and carries the same quantum numbers as the state $++$ living on the orbifold plane O.

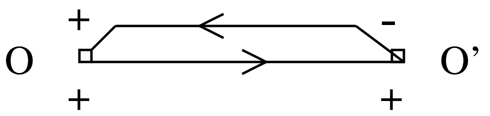

Figure 7: Superposition of the D-brane configurations of figs. 6(a), (g).

Since this configuration has total mass $2\pi R T_D$, we expect that for large R this configuration will be unstable under decay into the state $++$ living on the orbifold plane, as the latter has a finite mass independent of R. On the other hand, for small R the total mass of the system shown in Fig. 7 goes to zero and we expect the system to be stable. Thus if our interpretation of the tachyonic instability is correct, then for large R the system should have tachyonic modes, but for sufficiently small R there should be no tachyonic mode. This is indeed shown to be the case by analyzing the spectrum of open strings for this system.

Thus we arrive at the following scenario. For sufficiently small radius, the system shown in Fig. 7 is stable and describes the lowest mass configuration with these quantum numbers. As we increase R beyond a critical radius R_c, a tachyonic mode appears and the system becomes unstable due to the possibility of decaying into the $++$ state.[5] Thus the mass of the D-string anti- D-string system at $R = R_c$ must be equal to the mass m_{++} of the $++$ state living on the orbifold plane. This gives

$$m_{++} = 2\pi R_c T_D \,, \tag{1.1}$$

where T_D is the tension of the D-string. Measured in the string metric, $T_D = (2\pi\alpha' g)^{-1}$ where g is the string coupling constant. Explicit calculation shows that in this case $R_c = \sqrt{\alpha'/2}$. This gives

$$m_{++} = (\sqrt{2\alpha'}g)^{-1}. \tag{1.2}$$

will have zero energy. Since this configuration does not carry any charge and has the same quantum number as the vacuum, this is what we should expect.

[4]Non-BPS bound states in quantum field theory have been discussed recently in [14].

[5]In fact at $R = R_c$ the 'tachyon' becomes an exactly marginal operator in the underlying conformal field theory.

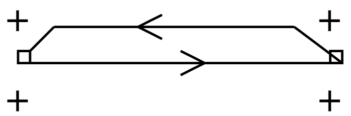

Figure 8: Superposition of the D-brane configurations of figs. 6(c), (g).

This result should be interpreted with care, since m_{++} may be affected by the presence of the second orbifold plane at a distance πR_c away, and hence the mass m_{++} calculated this way need not be the same as the mass m_{++} of a doubly charged state living on an isolated orbifold plane. Although we do not have a direct knowledge of the R dependence of m_{++}, we have indirect evidence that in the $g \to 0$ limit m_{++} might indeed be independent of R. This is seen by considering the superposition of the D-string configurations shown in figs. 6(c) and 6(g), as shown in Fig. 8. Again we find that for small R there is no tachyon, whereas for large R there is a tachyon signalling the decay of this state into a $++$ state living on the first orbifold plane, and a $++$ state living on the second orbifold plane.

The critical radius where the tachyon appears turns out to be $R_c' = \sqrt{2\alpha'}$, and hence the total mass of the pair of D-strings at this critical radius is given by $2\pi R_c' T_D = (g\sqrt{\alpha'/2})^{-1}$. This should be equal to $2m_{++}$.[6] This gives the same value of m_{++} as given in eq.(1.2) although now the separation between the two orbifold planes is double of what it was in the previous case. Since m_{++} calculated this way has the same value for $R = R_c$ and $R = R_c'$, we have reason to believe that it might be independent of R and gives the value of m_{++} for $R \to \infty$.

Given the answer (1.2) for m_{++} one can reexpress it in terms of the variables of the dual theory. If $\widetilde{g}$ is the coupling constant of the dual theory, then the mass of this state, measured in the string metric of the dual theory, turns out to be:

$$\widetilde{m}_{++} = (\sqrt{2\alpha'})^{-1} \widetilde{g}^{1/2}. \tag{1.3}$$

This will be the mass of a $++$ state living on the coincident D5-brane $-$ O5-plane system in the limit of large $\widetilde{g}$.

2. D-string ending on an orbifold plane

In this section we shall construct the boundary states describing D-strings ending on an orbifold plane for each of the four configurations displayed in Fig. 3, as well as their superposition shown in Fig. 4. We begin in subsection 2.1 by reviewing the construction of the boundary state describing a D-string in type IIB string theory in flat ten dimensional Minkowski space. In subsection 2.2 we extend this construction to a D-string ending on the orbifold plane. In subsection 2.3 we analyze the dynamics of a configuration obtained by superposition of two such D-string configurations.

[6] Since there are a pair of $++$ states, one living on each orbifold, there will also be a contribution to the mass due to the interaction between these two states. However, since each state has mass of order g^{-1} and the Newton's constant is of order g^2, this interaction energy will be of order unity, and will be negligible compared to the total mass of the system in the $g \to 0$ limit.

2.1. Review of D-strings in flat space

Let us consider a configuration of D-string in type IIB string theory on $R^{9,1}$. We shall denote the coordinates of $R^{9,1}$ by $x^0, \ldots x^9$, and take the D-string to be stretched along x^9 and situated at $x^1 = \ldots x^8 = 0$. A convenient description of the D-string is given in terms of boundary states describing the wave-function of a closed string emitted from the D-string[7, 8, 9, 6]. We work in the notation of [6] by making a double wick rotation $x^0 \to ix^0$, $x^1 \to ix^1$, so that the directions tangential to the D-string world-sheet are space-like, and then go to the light-cone gauge by taking $x^1 \pm x^2$ as the light-cone directions.[7] We now define the coherent states:

$$|k,\eta\rangle_{\substack{NSNS\\RR}} = \exp\Big(\sum_{n=1}^{\infty}[-\frac{1}{n}\sum_{\mu=0,9}\alpha^\mu_{-n}\tilde{\alpha}^\mu_{-n} + \frac{1}{n}\sum_{\mu=3}^{8}\alpha^\mu_{-n}\tilde{\alpha}^\mu_{-n}] + \\ + i\eta\sum_{r>0}[-\sum_{\mu=0,9}\psi^\mu_{-r}\tilde{\psi}^\mu_{-r} + \sum_{\mu=3}^{8}\psi^\mu_{-r}\tilde{\psi}^\mu_{-r}]\Big)|k,\eta\rangle^{(0)}_{\substack{NSNS\\RR}}, \tag{2.1}$$

where k denotes the eight dimensional momentum $(k^1, \ldots k^8)$ transverse to the D-string, α^μ_n and $\tilde{\alpha}^\mu_n$ are the right- and the left-moving modes of the bosonic coordinate X^μ, ψ^μ_r and $\tilde{\psi}^\mu_r$ are the modes of the right- and the left-moving fermionic coordinates ψ^μ, $\tilde{\psi}^\mu$, and η can take values ± 1. Our notations and normalization conventions are described in appendix A; also we have set the string tension to $1/2\pi$ ($\alpha' = 1$). The sum over n runs over positive integers, whereas the sum over r runs over positive integers (integers $+\frac{1}{2}$) in the RR (NSNS) sector. $|k,\eta\rangle^{(0)}$ denotes the Fock vacuum carrying momentum k in directions transverse to the D-string and zero momentum along directions tangential to the D-string. In the NSNS sector this uniquely specifies this state and hence it is independent of η. The situation in the RR sector is a bit more complicated due to the presence of the fermionic zero modes. In order to define $|k,\eta\rangle^{(0)}_{RR}$ we define

$$\psi^\mu_\pm = \frac{1}{\sqrt{2}}(\psi^\mu_0 \pm i\tilde{\psi}^\mu_0), \qquad \mu = 0, 3, 4 \ldots 9, \tag{2.2}$$

where ψ^μ_0 and $\tilde{\psi}^\mu_0$ are the zero modes of the left- and the right-moving modes of the world-sheet fermions, normalized as in appendix A. We now define $|k,-\rangle^{(0)}_{RR}$ to be the RR ground state satisfying:

$$\begin{aligned} \psi^\mu_-|k,-\rangle^{(0)}_{RR} &= 0 \qquad \text{for} \quad \mu = 0, 9 \\ \psi^\mu_+|k,-\rangle^{(0)}_{RR} &= 0 \qquad \text{for} \quad \mu = 3, 4 \ldots 8\,. \end{aligned} \tag{2.3}$$

$|k,+\rangle^{(0)}_{RR}$ is now defined as

$$|k,+\rangle^{(0)}_{RR} = \prod_{\mu=3}^{8}\psi^\mu_- \prod_{\mu=0,9}\psi^\mu_+\,|k,-\rangle^{(0)}_{RR}\,. \tag{2.4}$$

[7]The advantage of the light-cone gauge is that we avoid having to deal with the ghost sector. However since the treatment of this sector for D-branes and the usual string theory is identical, we could easily work in the fully covariant formulation following [7, 8, 9].

Let us now define

$$|\eta\rangle_{NSNS} = \mathcal{N}\int \Big(\prod_{\mu=1}^{8} dk^\mu\Big)\, |k,\eta\rangle_{NSNS}\,,$$
$$|\eta\rangle_{RR} = 4i\mathcal{N}\int \Big(\prod_{\mu=1}^{8} dk^\mu\Big)\, |k,\eta\rangle_{RR}\,, \tag{2.5}$$

$$|U\rangle_{NSNS} = \frac{1}{\sqrt{2}}(|+\rangle_{NSNS} - |-\rangle_{NSNS})\,,$$
$$|U\rangle_{RR} = \frac{1}{\sqrt{2}}(|+\rangle_{RR} + |-\rangle_{RR})\,, \tag{2.6}$$

$$|D1\rangle = \frac{1}{\sqrt{2}}(|U\rangle_{NSNS} + |U\rangle_{RR})\,,$$
$$|\bar{D}1\rangle = \frac{1}{\sqrt{2}}(|U\rangle_{NSNS} - |U\rangle_{RR})\,. \tag{2.7}$$

In eqs.(2.6) and (2.7) the symbol U stands for untwisted sector; although at present we are dealing with D-strings in flat space, we have attached these symbols in anticipation of the analysis in the next subsection. $\mathcal{N}$ is a normalization constant which will be fixed later. The states $|\eta\rangle_{NSNS}$ and $|\eta\rangle_{RR}$ defined in eq.(2.5) can be shown to satisfy the boundary conditions relevant for a D-string situated at $x^\mu = 0$ $(1 \le \mu \le 8)$:

$$\begin{aligned}
X^\mu(\tau=0,\sigma)|\eta\rangle_{\substack{NSNS\\RR}} &= 0 \qquad \text{for} \quad 3 \le \mu \le 8\,,\\
\partial_\tau X^\mu(\tau=0,\sigma)|\eta\rangle_{\substack{NSNS\\RR}} &= 0 \qquad \text{for} \quad \mu = 0, 9\,,\\
(\psi^\mu - i\eta\widetilde{\psi}^\mu)|\eta\rangle_{\substack{NSNS\\RR}} &= 0 \qquad \text{for} \quad 3 \le \mu \le 8\,,\\
(\psi^\mu + i\eta\widetilde{\psi}^\mu)|\eta\rangle_{\substack{NSNS\\RR}} &= 0 \qquad \text{for} \quad \mu = 0, 9\,,\\
x^\mu|\eta\rangle_{\substack{NSNS\\RR}} &= 0 \qquad \text{for} \quad \mu = 1, 2\,.
\end{aligned} \tag{2.8}$$

The states $|U\rangle_{NSNS}$ and $|U\rangle_{RR}$ defined in eq.(2.6) satisfy GSO projection, *i.e.* they are invariant under the operators $(-1)^F$ and $(-1)^{\widetilde{F}}$ defined in appendix A. Finally, the states $|D1\rangle$ and $|\bar{D}1\rangle$ defined in eq.(2.7) denote the boundary states associated with a D-string and an anti-D-string respectively. (An anti-D-string can be identified with a D-string with opposite orientation.) Note that although the states $|U\rangle_{NSNS}$ and $|U\rangle_{RR}$ by themselves satisfy the GSO projection as well as the boundary condition (2.8), we need to take the specific linear combinations defined in eq.(2.7) for satisfying the open-closed consistency condition[6] that the amplitude for emission and reabsorption of a closed string from the brane must be interpretable as the partition function of open string states satisfying appropriate GSO projection. We shall now see explicitly how this happens.

The amplitude for the emission and reabsorption of a closed string from the D-string is given by

$$\int_0^\infty dl\, \langle D1|e^{-lH_c}|D1\rangle\,, \tag{2.9}$$

where H_c is the hamiltonian for closed strings in the light-cone gauge:

$$H_c = \pi p^2 + 2\pi \sum_{\mu=0,3,\dots 9} \Big[\sum_{n=1}^{\infty} (\alpha^{\mu}_{-n}\alpha^{\mu}_{n} + \tilde{\alpha}^{\mu}_{-n}\tilde{\alpha}^{\mu}_{n}) + \sum_{r>0} r(\psi^{\mu}_{-r}\psi^{\mu}_{r} + \tilde{\psi}^{\mu}_{-r}\tilde{\psi}^{\mu}_{r}) \Big] + 2\pi C_c\,. \quad (2.10)$$

The constant C_c takes the value -1 in the NSNS sector, and zero in the RR sector. (2.9) can be evaluated with the help of the following identities:[8,9]

$$\begin{aligned}
{}_{NSNS}\langle +|e^{-lH_c}|+\rangle_{NSNS} = {}_{NSNS}\langle -|e^{-lH_c}|-\rangle_{NSNS} &= \mathcal{N}^2 l^{-4}\frac{f_3(q)^8}{f_1(q)^8}\,,\\
{}_{NSNS}\langle +|e^{-lH_c}|-\rangle_{NSNS} = {}_{NSNS}\langle -|e^{-lH_c}|+\rangle_{NSNS} &= \mathcal{N}^2 l^{-4}\frac{f_4(q)^8}{f_1(q)^8}\,,\\
{}_{RR}\langle +|e^{-lH_c}|+\rangle_{RR} = {}_{RR}\langle -|e^{-lH_c}|-\rangle_{RR} &= -\mathcal{N}^2 l^{-4}\frac{f_2(q)^8}{f_1(q)^8}\,,\\
{}_{RR}\langle +|e^{-lH_c}|-\rangle_{RR} = {}_{RR}\langle -|e^{-lH_c}|+\rangle_{RR} &= 0\,, \qquad (2.11)
\end{aligned}$$

where $q \equiv \exp(-2\pi l)$ and[7],

$$\begin{aligned}
f_1(q) &= q^{1/12}\prod_{n=1}^{\infty}(1-q^{2n})\,,\\
f_2(q) &= \sqrt{2}\,q^{1/12}\prod_{n=1}^{\infty}(1+q^{2n})\,,\\
f_3(q) &= q^{-1/24}\prod_{n=1}^{\infty}(1+q^{2n-1})\,,\\
f_4(q) &= q^{-1/24}\prod_{n=1}^{\infty}(1-q^{2n-1})\,. \qquad (2.12)
\end{aligned}$$

The functions f_i have the following modular transformation properties:

$$\begin{aligned}
f_1(e^{-\pi/t}) = \sqrt{t} f_1(e^{-\pi t})\,, &\qquad f_2(e^{-\pi/t}) = f_4(e^{-\pi t})\,,\\
f_3(e^{-\pi/t}) = f_3(e^{-\pi t})\,, &\qquad f_4(e^{-\pi/t}) = f_2(e^{-\pi t})\,. \qquad (2.13)
\end{aligned}$$

Let H_o be the Hamiltonian for an open string with ends lying on the D-brane

$$H_o = \pi p^2 + \pi \sum_{\mu=0,3,\dots 9} \Big[\sum_{n=1}^{\infty} \alpha^{\mu}_{-n}\alpha^{\mu}_{n} + \sum_{r>0} r\psi^{\mu}_{-r}\psi^{\mu}_{r} \Big] + \pi C_o\,. \quad (2.14)$$

C_o vanishes in the Ramond sector and takes the value $-1/2$ in the NS sector. p denotes the open string momentum tangential to the D-string. Let us denote by Tr_{NS} the trace over the NS sector states of the open string Hilbert space and Tr_R the trace over the Ramond sector states of the open string Hilbert space. These traces include

[8] The factor of l^{-4} in these equations comes from momentum integration.

[9] In defining the bra vectors we use the convention that the factor of i appearing in eq.(2.5) is not conjugated.

an integration over the momentum tangential to the D-string, weighted by the density of states $A/4\pi^2$, where A is the (infinite) area of the $x^0 - x^9$ plane. *Also both traces are taken without the GSO projection.* Defining new variables $t = (2l)^{-1}$, $\tilde{q} = e^{-\pi t}$, and using (2.11)-(2.14) we get

$$\int_0^\infty dl \ {}_{NSNS}\langle +|e^{-lH_c}|+\rangle_{NSNS} = \int_0^\infty dl \ {}_{NSNS}\langle -|e^{-lH_c}|-\rangle_{NSNS}$$
$$= 8\mathcal{N}^2 \int_0^\infty \frac{dt}{t^2}\Big(\frac{f_3(\tilde{q})}{f_1(\tilde{q})}\Big)^8 = \int_0^\infty \frac{dt}{2t} Tr_{NS}(e^{-2tH_o}), \quad (2.15)$$

$$\int_0^\infty dl \ {}_{NSNS}\langle +|e^{-lH_c}|-\rangle_{NSNS} = \int_0^\infty dl \ {}_{NSNS}\langle -|e^{-lH_c}|+\rangle_{NSNS}$$
$$= 8\mathcal{N}^2 \int_0^\infty \frac{dt}{t^2}\Big(\frac{f_2(\tilde{q})}{f_1(\tilde{q})}\Big)^8 = \int_0^\infty \frac{dt}{2t} Tr_{R}(e^{-2tH_o}), \quad (2.16)$$

$$\int_0^\infty dl \ {}_{RR}\langle +|e^{-lH_c}|+\rangle_{RR} = \int_0^\infty dl \ {}_{RR}\langle -|e^{-lH_c}|-\rangle_{RR}$$
$$= -8\mathcal{N}^2 \int_0^\infty \frac{dt}{t^2}\Big(\frac{f_4(\tilde{q})}{f_1(\tilde{q})}\Big)^8 = \int_0^\infty \frac{dt}{2t} Tr_{NS}(e^{-2tH_o}(-1)^F), \quad (2.17)$$

$$\int_0^\infty dl \ {}_{RR}\langle +|e^{-lH_c}|-\rangle_{RR} = \int_0^\infty dl \ {}_{RR}\langle -|e^{-lH_c}|+\rangle_{RR} = 0\,, \quad (2.18)$$

provided we choose

$$32\mathcal{N}^2 = \frac{A}{4\pi^2}\,. \quad (2.19)$$

The $-$ sign in eq.(2.17) shows that the open string NS sector ground state is odd under $(-1)^F$.

Using eqs. (2.6), (2.7), and (2.15)-(2.18) we can now evaluate (2.9):

$$\int_0^\infty dl\,\langle D1|e^{-lH_c}|D1\rangle = \int_0^\infty \frac{dt}{2t}\Big[Tr_{NS}\Big(e^{-2tH_o}\frac{1+(-1)^F}{2}\Big) - $$
$$-Tr_R\Big(e^{-2tH_o}\frac{1+(-1)^F}{2}\Big)\Big]\,, \quad (2.20)$$

where we have used the identity

$$Tr_R\Big(e^{-2tH_o}(-1)^F\Big) = 0\,. \quad (2.21)$$

(2.20) can be identified as the one loop partition function of open strings with both ends lying on the D-string.

Let us now consider a D-string anti-D-string pair situated at $x^1 = \ldots x^8 = 0$. This system is described by a boundary state:

$$|D1;\bar{D}1\rangle \equiv |D1\rangle + |\bar{D}1\rangle\,. \quad (2.22)$$

The corresponding amplitude for the emission and reabsorption of a closed string from this D-string pair is given by

$$\int_0^\infty dl\, \langle D1;\bar{D1}|e^{-lH_c}|D1;\bar{D1}\rangle\,. \tag{2.23}$$

This can be easily evaluated using eqs.(2.15)-(2.18) by noting that $|D1\rangle + |\bar{D1}\rangle = \sqrt{2}|U\rangle_{NSNS}$. However in order to give a physical interpretation of the result it is more convenient to rewrite this integral as

$$\int_0^\infty dl\, [\langle D1|e^{-lH_c}|D1\rangle + \langle \bar{D1}|e^{-lH_c}|\bar{D1}\rangle + \langle D1|e^{-lH_c}|\bar{D1}\rangle + \langle \bar{D1}|e^{-lH_c}|D1\rangle]\,. \tag{2.24}$$

Each of the first two terms are given by (2.20) and can be interpreted as the partition function of open strings with both ends lying on the D-string and both ends lying on the anti-D-string respectively. On the other hand, using eqs.(2.6), (2.7), (2.15)-(2.18), (2.21) we get

$$\begin{aligned}\int_0^\infty dl\, \langle D1|e^{-lH_c}|\bar{D1}\rangle &= \int_0^\infty dl\, \langle \bar{D1}|e^{-lH_c}|D1\rangle \\ &= \int_0^\infty \frac{dt}{2t} Tr_{NS-R}\Big(e^{-2tH_o}\frac{1-(-1)^F}{2}\Big)\,,\end{aligned} \tag{2.25}$$

where Tr_{NS-R} denotes the difference between the traces in the NS and the R sector of open strings. These have to be interpreted as the partition functions of open strings starting on the D-string and ending on the anti-D-string and vice versa. The $-$ sign in front of the $(-1)^F$ term in the partition function shows that these open strings have opposite GSO projection compared to the open strings with both ends ending on D-string or anti-D-string. Since the NS sector ground state has $(-1)^F = -1$, we see that it now survives the GSO projection, giving rise to the tachyonic mode living on the D-string - anti-D-string world-sheet[15, 16, 17].

We can formalise this by assigning 2×2 Chan-Paton matrices to the open strings living on the world volume of the D-string anti-D-string pair. The open strings with both ends lying on the D-string or anti-D-string correspond to diagonal Chan-Paton matrices $\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$ and $\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$, whereas the open strings with two ends lying on the two different branes correspond to off-diagonal Chan-Paton matrices $\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$ and $\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}$. The action of $(-1)^F$ on the Chan-Paton factor is taken to be conjugation by the matrix $\begin{pmatrix}1 & \\ & -1\end{pmatrix}$. With this convention, the total wave-function of any physical open string state is always invariant under $(-1)^F$.

2.2. D-string on a Z_2 orbifold

Let us now consider a Z_2 orbifold of the D-string configuration discussed in the previous section. The orbifold group is generated by the element

$$g = (-1)^{F_L}\cdot \mathcal{I}_4\,, \tag{2.26}$$

where $\mathcal{I}_4$ changes the sign of the coordinates $x^6, \dots x^9$, and $(-1)^{F_L}$ changes the sign of the Ramond sector ground state on the left without having any action on any of the oscillators, or the ground state of any other sector. Note that the D-string along the 9th direction is transverse to the orbifold fixed plane spanned by $x^1, \dots x^5$. Our first task will be to verify that the states $|U\rangle_{NSNS}$ and $|U\rangle_{RR}$ are invariant under g. In both, the NSNS and the RR sectors, $\mathcal{I}_4$ induces the transformation

$$\alpha_n^\mu \to -\alpha_n^\mu, \quad \tilde{\alpha}_n^\mu \to -\tilde{\alpha}_n^\mu, \quad \psi_r^\mu \to -\psi_r^\mu, \quad \tilde{\psi}_r^\mu \to -\tilde{\psi}_r^\mu, \qquad \text{for} \quad 6 \le \mu \le 9\,. \tag{2.27}$$

$\mathcal{I}_4$ acts on the Fock vacuum in the NSNS sector as

$$|k^0, \dots k^9\rangle^{(0)}_{NSNS} \to |k^0, \dots k^5, -k^6, \dots -k^9\rangle^{(0)}_{NSNS}\,. \tag{2.28}$$

On the other hand $(-1)^{F_L}$ has no action in the NSNS sector. Thus $|\eta\rangle_{NSNS}$ defined in eqs. (2.1), (2.5) and hence $|U\rangle_{NSNS}$ defined in eq.(2.6) is invariant under g.

The action of $\mathcal{I}_4$ on the RR sector ground state takes the form

$$|k^0, \dots k^9, \eta\rangle^{(0)}_{RR} \to \prod_{\mu=6}^{9} (\sqrt{2}\psi_0^\mu) \prod_{\mu=6}^{9} (\sqrt{2}\tilde{\psi}_0^\mu) |k^0, \dots k^5, -k^6, \dots -k^9, \eta\rangle^{(0)}_{RR}\,. \tag{2.29}$$

On the other hand $(-1)^{F_L}$ acting on the RR ground state changes its sign. Using these relations one can verify that $|U\rangle_{RR}$ defined in eq.(2.6) is also invariant under g.

Since both $|U\rangle_{NSNS}$ and $|U\rangle_{RR}$ are invariant under g, the boundary states $|D1\rangle$ and $|\bar{D}1\rangle$ defined in (2.7) are also invariant under g. However, this does not mean that the boundary state of a D-string (anti-D-string) stretched along the 9th direction is given by $|D1\rangle$ ($|\bar{D}1\rangle$). To see the reason, we note that the state $|D1\rangle$ satisfies eq.(2.20), but the correct boundary state $|D1'\rangle$ should give an answer where inside the trace we have a projection operator projecting onto the subspace of g invariant open string states:

$$\int_0^\infty dl\, \langle D1'| e^{-lH_c} |D1'\rangle = \int_0^\infty \frac{dt}{2t} Tr_{NS-R}\Big(e^{-2tH_o} \frac{1+(-1)^F}{2} \frac{1+g}{2} \Big)\,. \tag{2.30}$$

As we shall now show, the remedy lies in modifying the definitions of $|D1\rangle$, $|\bar{D}1\rangle$ by adding to them appropriate coherent states from the twisted sector satisfying equations analogous to (2.8). For this we define the analog of (2.1) in the twisted sector:

$$\begin{aligned}|k, \eta\rangle_{\substack{NSNS;T \\ RR;T}} = \exp\Big(\sum_{n>0} [-\frac{1}{n} \sum_{\mu=0,9} \alpha_{-n}^\mu \tilde{\alpha}_{-n}^\mu + \frac{1}{n} \sum_{\mu=3}^{8} \alpha_{-n}^\mu \tilde{\alpha}_{-n}^\mu] \\ + i\eta \sum_{r>0} [-\sum_{\mu=0,9} \psi_{-r}^\mu \tilde{\psi}_{-r}^\mu + \sum_{\mu=3}^{8} \psi_{-r}^\mu \tilde{\psi}_{-r}^\mu] \Big) |k, \eta\rangle^{(0)}_{\substack{NSNS;T \\ RR;T}}\,,\end{aligned} \tag{2.31}$$

where $k \equiv (k^1, \dots k^5)$ denotes the 5-dimensional momentum transverse to the D-string and tangent to the orbifold plane. In the NSNS sector the integers n and r take values

$$\begin{aligned} n \in\ & Z_+ && \text{for} \quad \mu = 0, 3, 4, 5 \\ \in\ & Z_+ - \tfrac{1}{2} && \text{for} \quad \mu = 6, 7, 8, 9 \\ r \in\ & Z_+ - \tfrac{1}{2} && \text{for} \quad \mu = 0, 3, 4, 5 \\ \in\ & Z_+ && \text{for} \quad \mu = 6, 7, 8, 9\,. \end{aligned} \tag{2.32}$$

On the other hand, in the RR sector

$$\begin{aligned} n, r \in \quad & Z_+ && \text{for} \quad \mu = 0,3,4,5 \\ \in \ & Z_+ - \tfrac{1}{2} && \text{for} \quad \mu = 6,7,8,9\,. \end{aligned} \tag{2.33}$$

Here Z_+ denotes the set of positive integers.

Note that there are fermion zero modes in both the NSNS and the RR sector. Hence we must carefully define the states $|k,\eta\rangle^{(0)}_{\substack{NSNS;T\\RR;T}}$ in both sectors. In the NSNS sector the fermion zero modes are ψ^μ_0 and $\widetilde{\psi}^\mu_0$ for $6 \le \mu \le 9$. We define $\psi^\mu_\pm$ as in eq.(2.2) for these values of μ, and the ground state $|k,-\rangle^{(0)}_{NSNS;T}$ satisfies

$$\begin{aligned} \psi^\mu_-|k,-\rangle^{(0)}_{NSNS;T} &= 0 \qquad \text{for} \quad \mu = 9 \\ \psi^\mu_+|k,-\rangle^{(0)}_{NSNS;T} &= 0 \qquad \text{for} \quad \mu = 6,7,8\,. \end{aligned} \tag{2.34}$$

$|k,+\rangle^{(0)}_{NSNS;T}$ is now defined as

$$|k,+\rangle^{(0)}_{NSNS;T} = \Big(\prod_{\mu=6}^{8} \psi^\mu_-\Big)\, \psi^9_+|k,-\rangle^{(0)}_{NSNS;T}\,. \tag{2.35}$$

In the RR sector the fermion zero modes are ψ^μ_0, $\widetilde{\psi}^\mu_0$ for $\mu = 0,3,4,5$. We again define $\psi^\mu_\pm$ for these values of μ using eq.(2.2) and the ground state $|k,-\rangle^{(0)}_{RR;T}$ satisfies

$$\begin{aligned} \psi^\mu_-|k,-\rangle^{(0)}_{RR;T} &= 0 \qquad \text{for} \quad \mu = 0 \\ \psi^\mu_+|k,-\rangle^{(0)}_{RR;T} &= 0 \qquad \text{for} \quad \mu = 3,4,5\,. \end{aligned} \tag{2.36}$$

$|k,+\rangle^{(0)}_{RR;T}$ is defined as

$$|k,+\rangle^{(0)}_{RR;T} = \Big(\prod_{\mu=3}^{5} \psi^\mu_-\Big)\, \psi^0_+|k,-\rangle^{(0)}_{RR;T}\,. \tag{2.37}$$

We now define

$$|\eta\rangle_{NSNS;T} = 2\widetilde{\mathcal{N}} \int \Big(\prod_{\mu=1}^{5} dk^\mu\Big)|k,\eta\rangle_{NSNS;T}\,, \tag{2.38}$$

$$|\eta\rangle_{RR;T} = 2i\widetilde{\mathcal{N}} \int \Big(\prod_{\mu=1}^{5} dk^\mu\Big)|k,\eta\rangle_{RR;T}\,, \tag{2.39}$$

where $\widetilde{\mathcal{N}}$ is a suitable normalisation constant to be determined later.

The action of the GSO operators $(-1)^F$ and $(-1)^{\widetilde{F}}$ in the ground state of the NSNS sector are given by

$$(-1)^F: \quad \prod_{\mu=6}^{9}(\sqrt{2}\psi^\mu_0), \qquad (-1)^{\widetilde{F}}: \quad -\prod_{\mu=6}^{9}(\sqrt{2}\widetilde{\psi}^\mu_0)\,. \tag{2.40}$$

On the other hand, acting on the ground state of the RR sector these operators take the form:

$$(-1)^F: \quad \Big(\prod_{\mu=3}^{5}(\sqrt{2}\psi_0^\mu)\Big)(\sqrt{2}\psi_0^0), \qquad (-1)^{\widetilde{F}}: \quad -\Big(\prod_{\mu=3}^{5}(\sqrt{2}\widetilde{\psi}_0^\mu)\Big)(\sqrt{2}\widetilde{\psi}_0^0). \tag{2.41}$$

The overall sign in the definition of each operator is determined by requiring consistency with the residual space-time supersymmetry of the orbifold theory. With this choice of sign, the massless states coming from the NSNS sector transform as a vector under the 'internal' R-symmetry group SO(4) acting on $x^6, \ldots x^9$, whereas the massless states coming from the RR sector transform as a vector under the rotation group SO(4) acting on x^0, x^3, x^4, x^5. This is precisely the bosonic part of the spectrum of a massless vector multiplet living on the orbifold plane. Using eqs. (2.31)-(2.41) we can verify that the following combinations of the boundary states are invariant under the GSO projection:

$$|T\rangle_{NSNS} = \frac{1}{\sqrt{2}}(|+\rangle_{NSNS;T} + |-\rangle_{NSNS;T}), \tag{2.42}$$

$$|T\rangle_{RR} = \frac{1}{\sqrt{2}}(|+\rangle_{RR;T} + |-\rangle_{RR;T}), \tag{2.43}$$

The closed string Hamiltonian in the twisted sector is given by eq.(2.10) with $C_c = 0$ in both NSNS and RR sector. The analog of eqs.(2.15)-(2.18) are now given by

$$\begin{aligned}\int_0^\infty dl \;\; {}_{NSNS;T}\langle+|e^{-lH_c}|+\rangle_{NSNS;T} &= \int_0^\infty dl \;\; {}_{NSNS;T}\langle-|e^{-lH_c}|-\rangle_{NSNS;T} \\ = 2^{3/2}\widetilde{\mathcal{N}}^2 \int_0^\infty \frac{dt}{t^{3/2}}\Big(\frac{f_4(\widetilde{q})f_3(\widetilde{q})}{f_1(\widetilde{q})f_2(\widetilde{q})}\Big)^4 &= \int_0^\infty \frac{dt}{2t} Tr_{NS}(e^{-2tH_o}\cdot g),\end{aligned} \tag{2.44}$$

$$\begin{aligned}\int_0^\infty dl \;\; {}_{NSNS;T}\langle+|e^{-lH_c}|-\rangle_{NSNS;T} &= \int_0^\infty dl \;\; {}_{NSNS;T}\langle-|e^{-lH_c}|+\rangle_{NSNS;T} \\ &= 0 = -\int_0^\infty \frac{dt}{2t} Tr_R(e^{-2tH_o}\cdot g),\end{aligned} \tag{2.45}$$

$$\begin{aligned}\int_0^\infty dl \;\; {}_{RR;T}\langle+|e^{-lH_c}|+\rangle_{RR;T} &= \int_0^\infty dl \;\; {}_{RR;T}\langle-|e^{-lH_c}|-\rangle_{RR;T} \\ = -2^{3/2}\widetilde{\mathcal{N}}^2 \int_0^\infty \frac{dt}{t^{3/2}}\Big(\frac{f_3(\widetilde{q})f_4(\widetilde{q})}{f_1(\widetilde{q})f_2(\widetilde{q})}\Big)^4 &= \int_0^\infty \frac{dt}{2t} Tr_{NS}(e^{-2tH_o}\cdot g\cdot(-1)^F),\end{aligned} \tag{2.46}$$

$$\begin{aligned}\int_0^\infty dl \;\; {}_{RR;T}\langle+|e^{-lH_c}|-\rangle_{RR;T} &= \int_0^\infty dl \;\; {}_{RR;T}\langle-|e^{-lH_c}|+\rangle_{RR;T} \\ &= 0 = -\int_0^\infty \frac{dt}{2t} Tr_R(e^{-2tH_o}\cdot g\cdot(-1)^F).\end{aligned} \tag{2.47}$$

provided we choose

$$2\widetilde{\mathcal{N}}^2 = \frac{L}{2\pi}, \tag{2.48}$$

where L is the (infinite) length of the x^0 direction. Conjugation by g changes the sign of the open string oscillators associated with the 6,7,8,9 directions.

There are some subtle points in eqs.(2.44)-(2.47) which we discuss below:

1. Although eqs.(2.44) and (2.46) have identical expressions up to a sign, various terms in these expressions have different origin in the two cases. For example, in (2.44) the factor of $(f_4(\tilde{q}))^4$ in the numerator comes from the fermions associated with the 6,7,8,9 directions, and the factor of $(f_3(\tilde{q}))^4$ in the numerator comes from the fermions associated with the 0,3,4,5 directions. In (2.46) their roles get reversed. This gives rise to different identifications of these expressions as partition functions in the open string sector, as reflected in eqs.(2.44) and (2.46). Similarly in eq.(2.45) the vanishing of the integrand is due to the zero modes of fermions associated with 6,7,8,9 directions, whereas in eq.(2.47) it is due to the zero modes of fermions associated with 0,3,4,5 directions. This again leads to different identification of these expressions in the open string channel.

2. Since g reverses the sign of k^9 but leaves k^0 invariant, only states with $k^9 = 0$ contribute to the trace in the open string sector. The $-$ sign in eq.(2.46) shows that the open string NS sector ground state with $k^9 = 0$ is odd under $(-1)^F \cdot g$, and hence is even under g since it is already known to be odd under $(-1)^F$. (The same result is reflected in the $+$ sign in eq.(2.44).) More generaly g acts on a tachyonic open string state with arbitrary momentum (k^0, k^9) as

$$|k^0, k^9\rangle \to |k^0, -k^9\rangle \,. \tag{2.49}$$

Let us now define

$$|D1,+\rangle = \frac{1}{2}(|U\rangle_{NSNS} + |U\rangle_{RR} + |T\rangle_{NSNS} + |T\rangle_{RR})\,, \tag{2.50}$$

$$|D1,-\rangle = \frac{1}{2}(|U\rangle_{NSNS} + |U\rangle_{RR} - |T\rangle_{NSNS} - |T\rangle_{RR})\,, \tag{2.51}$$

$$|\bar{D}1,+\rangle = \frac{1}{2}(|U\rangle_{NSNS} - |U\rangle_{RR} - |T\rangle_{NSNS} + |T\rangle_{RR})\,, \tag{2.52}$$

$$|\bar{D}1,-\rangle = \frac{1}{2}(|U\rangle_{NSNS} - |U\rangle_{RR} + |T\rangle_{NSNS} - |T\rangle_{RR})\,. \tag{2.53}$$

Using eqs.(2.15)-(2.18), (2.44)-(2.47) we now get

$$\int_0^\infty dl\,\langle D1,+|e^{-lH_c}|D1,+\rangle = \int_0^\infty \frac{dt}{2t} Tr_{NS-R}\Big(e^{-2tH_o}\frac{1+(-1)^F}{2}\frac{1+g}{2}\Big)\,, \tag{2.54}$$

where Tr_{NS-R} denotes the difference between the traces over the NS and R sector states. This expression is identical to the one in (2.30), showing that the $|D1,+\rangle$ state as defined above does represent a consistent boundary state in this orbifold theory. Similarly one can show that each of the other three states defined in (2.51)-(2.53) also represent consistent boundary states.

The interpretation of these four boundary states can be read out from their expressions. The sign of $|U\rangle_{RR}$ determines the sign of the charge carried by the state under the untwisted sector RR gauge field. Thus if this sign is positive then the state represents a D-string, whereas if it is negative then the state represents an anti-D-string. On the other hand the sign of $|T\rangle_{RR}$ determines the charge carried by the state under the twisted sector RR gauge field living on the orbifold plane. Thus the states given in eqs.(2.50)-(2.53) represent the configurations shown in 3(a), 3(c), 3(d) and 3(b) respectively.

2.3. Superposition of D-strings on orbifold

We shall now consider the superposition of the D-string configurations given in Fig. 3(a) and 3(d) as shown in Fig. 4. The corresponding boundary state is given by

$$|D1,+;\bar{D}1,+\rangle = |D1,+\rangle + |\bar{D}1,+\rangle\,. \tag{2.55}$$

Thus the amplitude for the emission and reabsorption of a closed string from this system is given by

$$\begin{aligned}\int_0^\infty dl\,\langle D1,+;\bar{D}1,+|e^{-lH_c}|D1,+;\bar{D}1,+\rangle &= \\ = \int_0^\infty dl\,[\langle D1,+|e^{-lH_c}|D1,+\rangle &+ \langle\bar{D}1,+|e^{-lH_c}|\bar{D}1,+\rangle + \\ &+\langle D1,+|e^{-lH_c}|\bar{D}1,+\rangle + \langle\bar{D}1,+|e^{-lH_c}|D1,+\rangle]\,.\end{aligned} \tag{2.56}$$

Each of the first two terms are given by eq.(2.54) and can be interpreted as the partition function for open string with both ends lying on the D1-brane or both ends lying on the $\bar{D}1$-brane. On the other hand each of the last two terms gives

$$\int_0^\infty \frac{dt}{2t} Tr_{NS-R}\Big(e^{-2tH_o}\frac{1-(-1)^F}{2}\frac{1-g}{2}\Big)\,. \tag{2.57}$$

These can be regarded as the partition functions of open string with one end lying on the D1-brane and the other end lying on the $\bar{D}1$-brane. The $-$ sign in front of the $(-1)^F$ term shows that these states have opposite GSO projection, and the $-$ sign in front of g shows that these states also have opposite g projection compared to open strings with both ends lying on the same brane. Due to the opposite $(-1)^F$ projection, the tachyon from the NS sector ground state survives. But comparison with eq.(2.49) shows that the action of g on the ground state now takes the form:

$$|k^0,k^9\rangle \to -|k^0,-k^9\rangle\,. \tag{2.58}$$

Thus the $k^9=0$ state is projected out and the g invariant tachyonic state takes the form:

$$|k^0,k^9\rangle - |k^0,-k^9\rangle\,. \tag{2.59}$$

If $T(x^0,x^9)$ denotes the tachyon field on the D-string in the position space representation, then (2.59) translates to the following boundary condition on the tachyon field:

$$T(x^0,x^9)=0 \qquad \text{at} \qquad x^9=0\,. \tag{2.60}$$

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Thus the minimal energy configuration of this system after tachyon condensation is expected to be described by a tachyon field configuration of the form described in Fig. 5.

3. D-strings in type IIB on $(R^{8,1} \times S^1)/Z_2$

In this section we shall analyse D-string on an orbifold $(R^{8,1} \times S^1)/Z_2$, with the D-string stretched along S^1, and the Z_2 transformation acting by the simultaneous action of $(-1)^{F_L}$ and the reversal of sign of the coordinate of S^1 and three of the coordinates of $R^{8,1}$. We begin in subsection 3.1 by constructing the boundary state describing D-string stretched along S^1 before the Z_2 modding. In subsection 3.2 we discuss the result of taking the Z_2 orbifold of this configuration, and construct the boundary states corresponding to all the eight configurations displayed in Fig. 6. In subsection 3.3 we consider appropriate superposition of these states to get the configurations of Fig. 7 and 8, and find in each case the critical radius at which the tachyon becomes massless.

3.1. D-string on $R^{8,1} \times S^1$

The analysis in this section is similar to that in section 2.1. We take the D-string to lie along the x^9 direction and take this direction to be compact with period $2\pi R$. The main difference from the analysis of section 2.1 arises due to the fact that in this case there is a new quantum number, – the winding number along S^1. We shall refer to the integer labelling the winding number as w_9. We now define new coherent states:

$$\begin{aligned}|k, w_9, \eta\rangle_{\substack{NSNS\\RR}} = \exp\Big(\sum_{n=1}^{\infty}[-\frac{1}{n}\sum_{\mu=0,9}\alpha^{\mu}_{-n}\tilde{\alpha}^{\mu}_{-n} + \frac{1}{n}\sum_{\mu=3}^{8}\alpha^{\mu}_{-n}\tilde{\alpha}^{\mu}_{-n}] + \\ +i\eta\sum_{r>0}[-\sum_{\mu=0,9}\psi^{\mu}_{-r}\tilde{\psi}^{\mu}_{-r} + \sum_{\mu=3}^{8}\psi^{\mu}_{-r}\tilde{\psi}^{\mu}_{-r}]\Big)|k, w_9, \eta\rangle^{(0)}_{\substack{NSNS\\RR}}, \end{aligned} \tag{3.1}$$

where the vacua $|k, w_9, \eta\rangle^{(0)}$ in the NSNS and RR sectors are defined in a manner analogous to $|k, \eta\rangle^{(0)}$ as in section 2.1. We now take linear combinations of these states of the form:

$$|\theta, \eta\rangle_{NSNS} = \mathcal{N}\sum_{w_9} e^{i\theta w_9}\int\Big(\prod_{\mu=1}^{8} dk^{\mu}\Big)|k, w_9, \eta\rangle_{NSNS}, \tag{3.2}$$

and,

$$|\theta, \eta\rangle_{RR} = 4i\mathcal{N}\sum_{w_9} e^{i\theta w_9}\int\Big(\prod_{\mu=1}^{8} dk^{\mu}\Big)|k, w_9, \eta\rangle_{RR}, \tag{3.3}$$

$\mathcal{N}$ being the same normalization constant defined in (2.19). We now define

$$|\theta, U\rangle_{NSNS} = \frac{1}{\sqrt{2}}(|\theta, +\rangle_{NSNS} - |\theta, -\rangle_{NSNS}), \tag{3.4}$$

$$|\theta, U\rangle_{RR} = \frac{1}{\sqrt{2}}(|\theta, +\rangle_{RR} + |\theta, -\rangle_{RR}), \tag{3.5}$$

$$|\theta, D1\rangle = \frac{1}{\sqrt{2}}(|\theta, U\rangle_{NSNS} + |\theta, U\rangle_{RR}) \,, \tag{3.6}$$

$$|\theta, \bar{D1}\rangle = \frac{1}{\sqrt{2}}(|\theta, U\rangle_{NSNS} - |\theta, U\rangle_{RR}) \,. \tag{3.7}$$

In order to find a physical interpretation of the parameter θ, let us analyse the term

$$\int_0^\infty dl \, \langle\theta, D1|e^{-lH_c}|\theta', D1\rangle \,, \tag{3.8}$$

which should give us the partition function of the open strings with one leg on the string described by the state $|\theta, D1\rangle$ and the other leg on the string described by the state $|\theta', D1\rangle$. H_c now has an additional term $\pi R^2 w_9^2$ compared to (2.10). Upon evaluation following standard method, we find that the integrand acquires an extra factor of

$$\sum_{w_9} e^{iw_9(\theta'-\theta) - l\pi R^2 w_9^2} \,, \tag{3.9}$$

compared to the integrand of (2.20). Using a Poisson resummation, we can rewrite (3.9) as

$$\frac{1}{R\sqrt{l}} \sum_m e^{-2t\pi \frac{1}{R^2}\{m - \frac{\theta-\theta'}{2\pi}\}^2} \,, \tag{3.10}$$

where $t = (2l)^{-1}$ as usual. In this equation the sum over m runs over integers. Since $R\sqrt{l}$ can be rewritten as

$$R \int dk_9 e^{-2t\pi k_9^2} \,, \tag{3.11}$$

we see that the effect of multiplication by (3.10) is to replace (3.11) in the trace over open string states by,

$$\sum_{k_9 = \frac{1}{R}(m - \frac{\theta-\theta'}{2\pi})} e^{-2t\pi k_9^2} \,. \tag{3.12}$$

The reason for replacing the integral over k_9 by a discrete sum can be attributed to the fact that the 9th direction is compact. However, note that instead of the usual quantization of k_9 in units of $1/R$, here we have an additional constant shift by $(\theta - \theta')/2\pi R$. Thus the wave-functions of these states, instead of being periodic along x^9, pick up a phase of $\exp(-i(\theta-\theta'))$ as $x^9 \to x^9 + 2\pi R$. This can be attributed to the presence of Wilson lines θ and θ' associated with the U(1) gauge fields living on the two D-strings. Thus the parameter θ measures the Wilson line associated with the U(1) gauge field living on the D-string.

3.2. D-strings on $(R^{8,1} \times S^1)/Z_2$

We now take the orbifold of the configuration described in the previous section by the Z_2 group generated by $(-1)^{F_L}$ accompanied by the reversal of sign of x^6, x^7, x^8 and x^9. For our purpose it will be convenient to start with a circle of radius $2R$, and define two Z_2 transformations g_1 and g_2 as follows. g_1 correponds to the transformation

$$x^6 \to -x^6, \quad x^7 \to -x^7, \quad x^8 \to -x^8, \quad x^9 \to -x^9 \,, \tag{3.13}$$

accompanied by the transformation $(-1)^{F_L}$. On the other hand g_2 corresponds to the transformation

$$x^6 \to -x^6, \quad x^7 \to -x^7, \quad x^8 \to -x^8, \quad x^9 \to 2\pi R - x^9 \,, \tag{3.14}$$

accompanied by the transformation $(-1)^{F_L}$. Then $g_2 g_1$ generates

$$g_2 g_1 : \qquad x^9 \to x^9 + 2\pi R \,. \tag{3.15}$$

Thus modding out by the group generated by g_1 and g_2 gives us back a circle of radius R.

With this convention, the w_9 even states appearing in (3.2), (3.3) may be regarded as untwisted sector states, whereas the w_9 odd states belong to the sector twisted by $g_1 g_2$. Since the transformation g_1 reverses the sign of w_9 in $|k, w_9, \eta\rangle$, we see from (3.2), (3.3) that only the $|0,\eta\rangle_{\substack{NSNS\\RR}}$ and $|\pi,\eta\rangle_{\substack{NSNS\\RR}}$ states are invariant under the orbifold projection. As in section 2.2, we need to add twisted sector coherent states to (3.6) and (3.7) in order that the amplitude for emission and reabsorption of a closed string can be reexpressed as open string partition function with appropriate projection. Since there are two sets of twisted sector states, one living at $x^9 = 0$ and twisted by g_1, and the other living at $x^9 = \pi R$ and twisted by g_2, we now have two sets of states of the kind described in (2.42), (2.43): $|T_1\rangle_{\substack{NSNS\\RR}}$ and $|T_2\rangle_{\substack{NSNS\\RR}}$ defined through eqs.(2.31)-(2.43). Our task now is to find appropriate linear combinations of these states so that the amplitude for the emission and reabsorption of a closed string can be reexpressed as an open string partition function with appropriate projections. An additional consistency requirement comes from the fact that if the two ends of the open string are on D-strings carrying the same Wilson line (0 or π), then due to eq.(3.12) the wave-function is periodic in x^9, and hence by eq.(3.15) the action of g_1 and g_2 on the state must be identical. On the other hand if one end of the open string is on a D-string without Wilson line and the other end is on a D-string with Wilson line π, then the wave-function is anti-periodic in x^9 and hence the action of g_1 and g_2 on the state must differ by a sign.

We find eight consistent boundary states satisfying these conditions. They are labelled by three quantum numbers, the Wilson line θ which can take values 0 or π, and two other numbers ϵ_1 and ϵ_2 each of which can take values ± 1. The boundary states are given by

$$\begin{aligned} |\theta, \epsilon_1, \epsilon_2\rangle = {}& \frac{1}{2}(|\theta, U\rangle_{NSNS} + \epsilon_1 |\theta, U\rangle_{RR}) + \frac{1}{2\sqrt{2}} \epsilon_2 (|T_1\rangle_{NSNS} + \epsilon_1 |T_1\rangle_{RR}) + \\ & + \frac{1}{2\sqrt{2}} e^{i\theta} \epsilon_2 (|T_2\rangle_{NSNS} + \epsilon_1 |T_2\rangle_{RR}) \,. \end{aligned} \tag{3.16}$$

The various parameters have the following interpretation:

1. ϵ_1 determines the orientation of the D-string. We shall choose the convention that positive ϵ_1 means that the string is oriented towards left in Fig. 6, *i.e.* from $x^9 = \pi R$ towards $x^9 = 0$.

2. $\epsilon_1\epsilon_2$ denotes the charge carried by the twisted sector RR gauge field living at $x^9 = 0$.

3. $\epsilon_1\epsilon_2 e^{i\theta}$ denotes the charge carried by the twisted sector RR gauge field living at $x^9 = \pi R$.

Thus the eight different configurations displayed in Fig. 6 are described by the following sets of values of $(\theta, \epsilon_1, \epsilon_2)$:[10]

$$\begin{array}{ll} (a): & (\pi,+,+) \\ (b): & (0,+,-) \\ (c): & (0,+,+) \\ (d): & (\pi,+,-) \\ (e): & (\pi,-,-) \\ (f): & (0,-,+) \\ (g): & (0,-,-) \\ (h): & (\pi,-,+)\,. \end{array} \tag{3.17}$$

Using the definition (3.16) of the state $|\theta, \epsilon_1, \epsilon_2\rangle$ one can easily verify that

$$\int_0^\infty dl\, \langle \theta', \epsilon_1', \epsilon_2'| e^{-lH_c} |\theta, \epsilon_1, \epsilon_2\rangle =$$

$$= \int_0^\infty \frac{dt}{2t} Tr_{NS-R}\Big\{e^{-2tH_o} \frac{1+\epsilon_1\epsilon_1'(-1)^F}{2} \frac{1+\epsilon_2\epsilon_2' g_1}{2} \frac{1+\epsilon_2\epsilon_2' e^{i(\theta-\theta')} g_2}{2}\Big\}\,. \tag{3.18}$$

From this we see that g_1 and g_2 have the same (opposite) projection if $(\theta - \theta')$ equals zero (π), as required. The sum over k_9 in the trace runs over $m/2R$ for all integers m.

3.3. Superpositions of D-strings on $(R^{8,1} \times S^1)/Z_2$

Let us now consider the superposition of the configurations given in figs. 6(a) and 6(g). Our object of interest is the partition function of the open string states with two legs lying on the two different strings. Using eqs.(3.17) and (3.18) we see that this contribution is given by

$$\int_0^\infty \frac{dt}{2t} Tr_{NS-R}\Big\{e^{-2tH_o} \frac{1-(-1)^F}{2} \frac{1-g_1}{2} \frac{1+g_2}{2}\Big\}\,. \tag{3.19}$$

Since this gives projection onto states with $(-1)^F = -1$, the 'tachyon' of mass^2 $-(2\alpha')^{-1}$ from the NS sector ground state survives. However since we need to project onto $g_1 g_2 = -1$ states, the momentum k_9 is quantized as $(m+\frac{1}{2})R^{-1}$ with m integer, and hence the $k^9 = 0$ mode is projected out. Thus the effective mass of the lowest mode, carrying $k^9 = \pm\frac{1}{2R}$ is given by

$$m^2 = (k^9)^2 - \frac{1}{2\alpha'} = \frac{1}{4R^2} - \frac{1}{2\alpha'}\,. \tag{3.20}$$

[10] These configurations are T-dual to the fractionally charged D0-branes discussed in [18, 19, 20, 21]. An analogous phenomenon was discussed in [22].

From this we see that the tachyonic mode disappears for

$$R \leq \sqrt{\frac{\alpha'}{2}} \equiv R_c \,. \qquad (3.21)$$

Thus if we identify the total mass of the D-string anti-D-string pair at this value of R as m_{++}, we get

$$m_{++} = 2\pi T_D R_c = (\sqrt{2\alpha'} g)^{-1} \,. \qquad (3.22)$$

Let us also consider the superposition of figs. 6(c) and 6(g) as shown in Fig. 8. Using eqs.(3.17) and (3.18) we see that the partition function of the open string with two ends lying on the two different D-strings is now given by

$$\int_0^\infty \frac{dt}{2t} Tr_{NS-R}\Big\{e^{-2tH_o} \frac{1-(-1)^F}{2} \frac{1-g_1}{2} \frac{1-g_2}{2}\Big\} \,. \qquad (3.23)$$

Since this gives projection onto states with $(-1)^F = -1$, the 'tachyon' of mass^2 $-(2\alpha')^{-1}$ from the NS sector ground state survives as in the previous case. But since we now need to project onto $g_1 g_2 = 1$ states, the momentum is quantized as m/R with m integer. However, the $k_9 = 0$ mode is still projected out, as the states are required to be odd under the action of g_1 and g_2 which transform k_9 to $-k_9$. Thus the effective mass of the lowest mode, carrying $k^9 = \pm\frac{1}{R}$ is given by

$$m^2 = (k^9)^2 - \frac{1}{2\alpha'} = \frac{1}{R^2} - \frac{1}{2\alpha'} \,. \qquad (3.24)$$

From this we see that the tachyonic mode disappears for

$$R \leq \sqrt{2\alpha'} \equiv R'_c \,. \qquad (3.25)$$

At this critical radius, we expect the mass of the D-string anti-D-string pair to be equal to twice the mass of a ++ state living on the orbifold plane. This gives,

$$2m_{++} = 2\pi T_D R'_c = 2(\sqrt{2\alpha'} g)^{-1} \,. \qquad (3.26)$$

This is the same as eq.(3.22).

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A. Notations and normalization conventions

We work with $\alpha' = 1$, *i.e.* string tension equal to $1/2\pi$. By a double Wick rotation we make the coordinate x^0 space-like and the coordinate x^1 time-like, so that we can formulate the theory in the light-cone gauge with $x^1 \pm x^2$ as the light-cone coordinates. The light-cone gauge action for closed string is then given by:

$$\frac{1}{4\pi}\int d\tau \int_0^1 d\sigma \sum_{\mu=0,3\ldots 9} \Big(\partial_\tau X^\mu \partial_\tau X^\mu - \partial_\sigma X^\mu \partial_\sigma X^\mu + \\ + i\psi^\mu(\partial_\tau+\partial_\sigma)\psi^\mu + i\widetilde{\psi}^\mu(\partial_\tau-\partial_\sigma)\widetilde{\psi}^\mu\Big)\,, \tag{A.1}$$

where X^μ, ψ^μ and $\widetilde{\psi}^\mu$ are periodic functions of σ with period 1. The equations of motion derived from this action give the following mode expansion for the fields;

$$\begin{aligned} X^\mu(\tau,\sigma) &= x^\mu + 2\pi p^\mu \tau + \frac{i}{\sqrt{2}}\sum_{n\neq 0}\frac{1}{n}(\alpha_n^\mu e^{-2\pi i n(\tau-\sigma)} + \widetilde{\alpha}_n^\mu e^{-2\pi i n(\tau+\sigma)})\,, \\ \psi^\mu(\tau,\sigma) &= \sqrt{2\pi}\sum_r \psi_r^\mu e^{-2\pi i r(\tau-\sigma)}\,, \\ \widetilde{\psi}^\mu(\tau,\sigma) &= \sqrt{2\pi}\sum_r \widetilde{\psi}_r^\mu e^{-2\pi i r(\tau+\sigma)}\,, \end{aligned} \tag{A.2}$$

for $\mu = 0,3,4\ldots 9$. The sum over n runs over integers, whereas the sum over r runs over integers and (integers $+\ \frac{1}{2}$) in the Ramond (R) and Neveu-Schwarz (NS) sector respectively. Besides these oscillator modes, we also need to include the zero modes x^1, x^2, p^1, p^2 in our list of dynamical variables. The commutation relations satisfied by these various oscillators are as follows:

$$[\alpha_m^\mu, \alpha_n^\nu] = [\widetilde{\alpha}_m^\mu, \widetilde{\alpha}_n^\nu] = m\delta_{\mu\nu}\delta_{m+n,0}, \qquad \text{for} \quad \mu,\nu = 0,3,4\ldots 9\,, \tag{A.3}$$

$$[x^\mu, p^\nu] = i\eta^{\mu\nu} \qquad \text{for} \quad 0\le\mu,\nu\le 9, \tag{A.4}$$

$$\{\psi_r^\mu, \psi_s^\nu\} = \{\widetilde{\psi}_r^\mu, \widetilde{\psi}_s^\nu\} = \delta_{\mu\nu}\delta_{r+s,0}\,, \qquad \text{for} \quad \mu,\nu = 0,3,4\ldots 9\,. \tag{A.5}$$

Here $\eta^{\mu\nu}$ is a diagonal matrix with η^{11} equal to -1 and all the other diagonal entries equal to $+1$. All other (anti-) commutators vanish.

Physical states are required to be invariant under the GSO operators $(-1)^F$ and $(-1)^{\widetilde{F}}$, which change the sign of all the world-sheet fermion fields ψ^μ and $\widetilde{\psi}^\mu$ respectively. The ground state of the NS sector is taken to be odd under these operations. Due to the presence of the fermion zero modes the Ramond sector ground state is degenerate. We choose our convention such that the Ramond sector ground states $|\Omega\rangle_{L,R}$ satisfying

$$\prod_{\mu=0,3,4\ldots 9}(\sqrt{2}\psi_0^\mu)|\Omega\rangle_R = |\Omega\rangle_R\,, \qquad \prod_{\mu=0,3,4\ldots 9}(\sqrt{2}\widetilde{\psi}_0^\mu)|\Omega\rangle_L = |\Omega\rangle_L\,, \tag{A.6}$$

are even under $(-1)^F$ and $(-1)^{\widetilde{F}}$ respectively.

An open string with two ends lying on a D-string along the x^9 direction is described by the same action as (A.1). However, instead of being periodic in σ, X^μ's now satisfy Neumann (Dirichlet) boundary condition for $\mu = 0, 9$ ($\mu = 3, 4, ..8$). The mode expansions are given by

$$\begin{aligned}
X^\mu(\tau,\sigma) &= x^\mu + 2\pi p^\mu\tau + i\sqrt{2}\textstyle\sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu e^{-\pi i n\tau}\cos n\pi\sigma && \text{for } \mu = 0, 9\,, \\
&= i\sqrt{2}\textstyle\sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu e^{-\pi i n\tau}\sin n\pi\sigma && \text{for } \mu = 3, 4\ldots 8\,, \\
\psi^\mu(\tau,\sigma) &= \sqrt{\pi}\textstyle\sum_r \psi_r^\mu e^{-\pi i r(\tau-\sigma)} && \text{for } \mu = 0, 3, 4\ldots 9\,, \\
\widetilde{\psi}^\mu(\tau,\sigma) &= \sqrt{\pi}\textstyle\sum_r \psi_r^\mu e^{-\pi i r(\tau+\sigma)} && \text{for } \mu = 0, 9\,, \\
&= -\sqrt{\pi}\textstyle\sum_r \psi_r^\mu e^{-\pi i r(\tau+\sigma)} && \text{for } \mu = 3, 4\ldots 8\,,
\end{aligned} \tag{A.7}$$

The commutation relations of these oscillators are also given by eq.(A.3). Physical states are required to be invariant under the operator $(-1)^F$ which changes the sign of the oscillators ψ_r^μ. The ground state of the NS sector is again taken to be odd under $(-1)^F$, and the ground state of the Ramond sector satisfying

$$\prod_{\mu=0,3,4,\ldots 9} \psi_0^\mu|\Omega\rangle = |\Omega\rangle\,, \tag{A.8}$$

is taken to be even under $(-1)^F$.

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